Received: *September 7, 1998*, Accepted: *October 5, 1998*

Colour connection structure of (supersymmetric) QCD $(2 \to 2)$ processes

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Abstract: The colour connection structure of QCD $(2 \to 2)$ processes is discussed, with emphasis on its application to the supersymmetric 2 parton $\to$ 2 sparton processes, which are currently being implemented in the HERWIG Monte Carlo event generator. The procedure described by Marchesini and Webber is found to be inadequate, and a new method is proposed. However, this alteration is unlikely to significantly affect the theoretical predictions for soft gluon radiation. A complete list of supersymmetric QCD $2 \to 2$ matrix elements and their colour decompositions is presented.

Keywords: 1/N Expansion, Supersymmetric Standard Model, QCD.

JHEP10(1998)006

Contents

1. Introduction

The simulation of soft gluon radiation in hard (supersymmetric) QCD processes [1, 2, 3, 4] requires that the corresponding matrix elements be rearranged according to the colour connections (defined by the colour flows) in the process [1]. In brief, this is because the colour connections in the parent process determine the cones in which the soft gluons radiate and hence the hadronisation occurs.

This rearrangement of terms is automatic if there is a unique colour flow associated with the process, as is the case for the QCD $qq' \to qq'$ scattering whose colour flow is shown in fig. 1, but for more complex processes the procedure involves some ambiguity. The purpose of this paper is to discuss and analyse this ambiguity, to propose a consistent and practicable method, and to illustrate the application of this new method in supersymmetric QCD 2 parton → 2 sparton processes which are being implemented in the `HERWIG` Monte Carlo event generator [4, 5, 6].

1.1. Colour flows in QCD

The matrix elements for processes with more than one colour flow consist of the 'planar' terms and the 'nonplanar' terms. The planar terms are those with single colour flows and the nonplanar terms are those with no single colour flow. The nonplanar terms are always suppressed by some inverse powers of N_C.

The colour flows for the four distinct $2 \to 2$ QCD processes are shown in figs. 1–4. For concreteness, let us consider the QCD process $q\bar{q} \to gg$, for which the leading-order

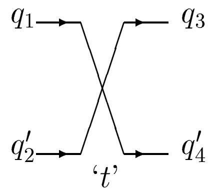

Figure 1: Colour flow for processes of the type $q_1 q_2' \to q_3 q_4'$.

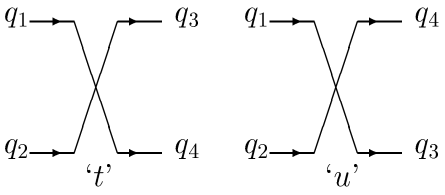

Figure 2: Colour flows for processes of the type $q_1 q_2 \to q_3 q_4$.

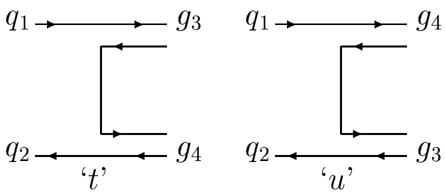

Figure 3: Colour flows for processes of the type $q_1 \bar{q}_2 \to g_3 g_4$.

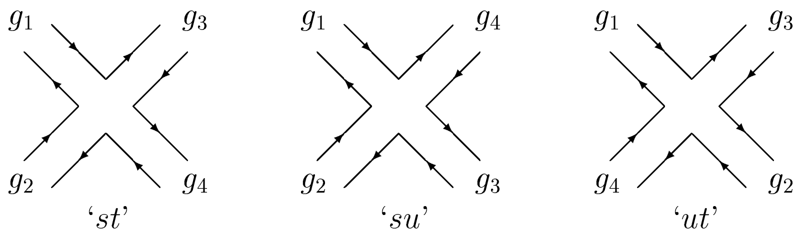

Figure 4: Colour flows for processes of the type $g_1 g_2 \to g_3 g_4$. The direction of colour flows (indicated by the arrows) is arbitrary.

spin- and colour-averaged matrix element squared is given by [7]:

$$g_s^4 C_F \frac{t^2+u^2}{s^2} \left[\left(\frac{u}{t}\right)_t + \left(\frac{t}{u}\right)_u + \left(-\frac{1}{N_C^2} \cdot \frac{s^2}{ut}\right)_{\text{n.p.}} \right]. \tag{1.1}$$

The first term in braces is a planar term corresponding to the t-channel colour flow, and the second term corresponds to the u-channel colour flow, as depicted in fig. 3. The third term, suppressed by the factor $(1/N_C^2)$, is a nonplanar contribution corresponding to a mixed colour flow, which needs more care in its treatment when we consider the rearrangement according to the colour connection.

Note that apart from the overall gauge invariance and positive definiteness of the matrix element squared, each of the above three terms is also gauge invariant and, in the case of the planar terms, positive definite. The gauge invariance follows from the fact that colour is formally an observable at Born approximation. As for the positive definiteness of the planar terms, this is obvious since the modulus squared of any part of the full matrix element must also be positive. This gauge invariance allows us to uniquely identify the planar terms and the nonplanar part in each process.

1.2. Radiation from colour connected partons

Let us recall the results of Ellis, Marchesini and Webber [1, 2] concerning the coherence of soft gluon radiations [8, 9] in hard QCD processes.

For each colour flow (each planar term) there is a cone around each incoming and outgoing parton direction bounded by the angle between the parton and the parton which is colour connected to it.[1] In the case of gluons (and gluinos) there are two such cones, one for the colour and one for the anticolour. The cones define the bounds for the soft gluons to be radiated (in the angular ordering approximation) and hence the hadronisation[2] to occur, to leading order in N_C.

As for the nonplanar part, this can be distributed in any way between the colour flows. Let us consider this in detail.

Returning to the example of $q_1\bar{q}_2 \to g_3 g_4$ introduced earlier, the radiation pattern, in the notation of [1], is

$$g_s^4 C_F \frac{t^2+u^2}{s^2}\left[\left(\frac{u}{t}\right)_t W_t + \left(\frac{t}{u}\right)_u W_u + \left(-\frac{1}{N_C^2}\cdot\frac{s^2}{ut}\right)_{\text{n.p.}} W_{\text{n.p.}}\right], \tag{1.2}$$

with

$$W_t = C_A\left[W_{34}+W_{13}+W_{24}-W_{12}\right]+2C_F W_{12}\,, \tag{1.3}$$

$$W_u = C_A\left[W_{34}+W_{14}+W_{23}-W_{12}\right]+2C_F W_{12}\,, \tag{1.4}$$

$$W_{\text{n.p.}} = C_A\left[W_{13}+W_{24}+W_{14}+W_{23}-2W_{12}\right]+2C_F W_{12}\,. \tag{1.5}$$

C_F and C_A are as usual the QCD colour factors $(N_C^2-1)/2N_C$ and (N_C) respectively. Terms proportional to C_A correspond to the radiation from the gluon legs, whereas the terms proportional to $2C_F$ correspond to the radiation from the quark legs. W_{ij} are the radiation functions for the parton pairs $\{i,j\}$ and are defined as:

$$W_{ij} = \frac{p_i\cdot p_j}{(p_i\cdot p_g)(p_j\cdot p_g)} \tag{1.6}$$

[1] This statement has not been explicitly verified in the supersymmetric case involving either long-lived spartons in the final state (the light gluino, whose existence is still controversial — see [10].) or short-lived ones in the intermediate state.

[2] When the (s)partons are massive, the analysis needs some modification [11]. For the heavy quark case, there will be a 'screening' of the collinear direction [3]. For the supersymmetric case, the analysis has not yet been carried out.

for the emission of a soft gluon of momentum p_g. In terms of the angles between parton pairs,

$$W_{ij} = \frac{1}{E_g^2} \cdot \frac{\xi_{ij}}{\xi_{ig}}\xi_{jg} \tag{1.7}$$

where $\xi_{ij} = 1 - \cos\theta_{ij} = p_i \cdot p_j / E_i E_j$. E_g is the soft gluon energy. This expression contains poles at $\theta_{ig} = 0$ and θ_{jg}. These correspond to the two collinear directions in gluon emission. Explicitly,

$$W_{ij} = W_{ij}^i + W_{ij}^j = \frac{1}{2E_g^2}\left\{\frac{\xi_{ij}}{\xi_{ig}\xi_{jg}} + \frac{1}{\xi_{ig}} - \frac{1}{\xi_{jg}}\right\} + \{i \leftrightarrow j\}. \tag{1.8}$$

W_{ij}^i have collinear singularities only at $\theta_{ig} = 0$. It can be shown that the azimuthal average of W_{ij}^i around the i-th parton direction is a step function with cut-off at θ_{ij}, hence the above claim concerning the radiation pattern.

We now consider modifying the planar terms such that the sum of them is equal to the original matrix element squared. Note that:

$$W_t - W_{\text{n.p.}} = C_A \left[W_{12} + W_{34} - W_{14} - W_{23}\right], \tag{1.9}$$

$$W_u - W_{\text{n.p.}} = C_A \left[W_{12} + W_{34} - W_{13} - W_{24}\right]. \tag{1.10}$$

In both expressions the radiation cancels in all four collinear directions, and so it is reasonable to approximate $W_{\text{n.p.}}$ by either W_t or W_u.

Hence it can be deduced that the radiation due to the $(1/N_C^2)$ suppressed nonplanar term can be treated approximately by distributing this term between the two colour flows in some ratio.

From the viewpoint of practicality in Monte Carlo simulations, this distribution should be such that the modified planar terms, which are called 'full terms' in [1], should be positive definite.

2. The MW procedure

The procedure of Marchesini and Webber (MW) for evaluating the full terms is as follows [1]:

- the full term should have the same pole structure and crossing symmetry as the planar term;
- it should remain positive definite in order to be interpreted as a probability distribution;
- the sum of the full terms should give the exact lowest order $2 \to 2$ matrix element squared.

The second and the third criteria above are essential as argued earlier. As for the pole structure, the introduction of an extra physical pole is bound to drive at least one of the full terms negative for some permutation of the external legs so this is a natural consequence of the second criterion. The requirement of correct crossing symmetry is never utilised in practice, and it is in fact too constraining to be practicable, as will be shown later. Discarding this requirement, we have essentially only the positivity and the sum to constrain the colour rearrangement, which is not sufficient to determine it uniquely. One might therefore introduce the following criterion, which is similar to the requirement of (not only physical) pole structure, and can be regarded as being inherent in the MW procedure:

- when a nonplanar term contains poles corresponding to two colour flows, this is split up by partial fractions, viz
$$\frac{1}{st}+\frac{1}{su}+\frac{1}{tu}=0 \tag{2.1}$$
for massless partons.

There are several disadvantages associated with this method:

- the decomposition of the nonplanar part is not unique without the additional criterion introduced above, and with this criterion the procedure sometimes fails and/or is still not unique, depending on the exact procedure by which the partial fractions are split up;
- the decomposition is not general under permutations of external legs;
- the decomposition becomes laborious when the number of colour flows is increased, and verification becomes an impossible task. The number of colour flows is a factorial function of the number of external partons.

The first two of these points are illustrated well for the case of the supersymmetric QCD process $gg \to \tilde{q}\tilde{q}^*$ which, at the tree level and in the massless limit, is described by the following matrix element squared:

$$\overline{|\mathcal{M}|^2}=\frac{g_s^4 N_C}{N_C^2-1}\cdot\frac{(u^2)_t+(t^2)_u+(-s^2/N_C^2)_{\mathrm{n.p.}}}{s^2}. \tag{2.2}$$

This does not have a unique rearrangement, not all such decompositions are positive, and many of them are not invariant under permutations of the external legs.

As a further illustration of this second point, consider the process $qq \to qq$, which has two colour flow structures as shown in fig. 2, and whose matrix element squared is rearranged, according to [1], as follows:

$$\overline{|\mathcal{M}|^2}=\frac{g_s^4 C_F}{N_C}\left[\left(\frac{s^2+u^2}{t^2}\right)+\left(\frac{s^2+t^2}{u^2}\right)+\left(-\frac{2}{N_C}\cdot\frac{s^2}{ut}\right)\right] \tag{2.3}$$

$$=\frac{g_s^4 C_F}{N_C}\left[\left(\frac{s^2+u^2}{t^2}+\frac{2}{N_C}\cdot\frac{s}{t}\right)+(u\leftrightarrow t)\right]. \tag{2.4}$$

This arrangement does not preserve the crossing symmetries $(s \leftrightarrow u)$ and $(s \leftrightarrow t)$ of the planar terms. It is in fact impossible to preserve the crossing symmetry while also preserving the pole structure, since the only dimensionless combination of s, t and u that is $(s \leftrightarrow u)$ symmetric is su/t^2. Taking into account the pole structure, the nonplanar term s^2/ut can not be expressed as a linear combination of su/t^2, st/u^2 and a constant.

Although (2.4) seems a natural rearrangement, it fails when $N_C = 2$ and the leg permutation $(s \leftrightarrow u)$ is made, corresponding to $q\bar{q} \to q\bar{q}$:

$$\overline{|\mathcal{M}|^2} = \frac{g_s^4 C_F}{N_C}\left[\left(\frac{s^2+u^2}{t^2} + \frac{2}{N_C}\cdot\frac{u}{t}\right) + \left(\frac{u^2+t^2}{s^2} + \frac{2}{N_C}\cdot\frac{u}{s}\right)\right]. \tag{2.5}$$

If we now set $N_C = 2$ in the second term above, we obtain

$$\frac{u^2+t^2}{s^2} + \frac{u}{s} = \frac{(-t)(u-t)}{s^2} \tag{2.6}$$

which is negative whenever $(u-t)$ is negative. For $N_C = 3$ the above expression remains positive but this is accidental.[3]

$$u^2 + t^2 + \frac{2}{3}su = \frac{1}{3}(u-t)^2 + \frac{2}{3}t^2 > 0. \tag{2.7}$$

One method for obtaining a positive definite decomposition is to split the nonplanar term as follows:

$$-\frac{u^2}{st} = \frac{u^2}{s-t}\left(-\frac{1}{t} + \frac{1}{s}\right). \tag{2.8}$$

Since both contributions are positive, the full terms are also positive. However, the fact remains that the decomposition is not universal under permutations of the external legs.

3. New procedure proposed

To solve the above problems, we propose the following procedure. Let the overall matrix element squared be given by

$$\overline{|\mathcal{M}|^2}_{tot} = \left(\sum_i \overline{|\mathcal{M}|^2}_i\right) + (\text{n.p.}) = \overline{|\mathcal{M}|^2}_{\text{planar}} + (\text{n.p.})\,. \tag{3.1}$$

where $\overline{|\mathcal{M}|^2}_i$ is the planar term for the i-th colour flow, and (n.p.) represents the nonplanar part. Then the i-th full term is given by

$$\overline{|\mathcal{M}|^2}_{\text{full},i} = \frac{\overline{|\mathcal{M}|^2}_i}{\overline{|\mathcal{M}|^2}_{\text{planar}}}\overline{|\mathcal{M}|^2}_{\text{tot}} \tag{3.2}$$

$$= \overline{|\mathcal{M}|^2}_i + \frac{\overline{|\mathcal{M}|^2}_i}{\overline{|\mathcal{M}|^2}_{\text{planar}}}(\text{n.p.})\,. \tag{3.3}$$

[3]In general, when a full term is positive for a certain N_C, one can prove trivially that it is also positive for all higher N_C since the planar term must always be positive.

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In other words, we split the nonplanar part by the ratios of the planar terms.

This is positive definite, as is obvious in equation (3.2). This method resolves all three drawbacks of the MW method listed above, and carries some additional advantages:

- the correct pole structure in each full term is automatically ensured — the larger the planar term the larger the full term. No extra pole is introduced since the sum of the planar terms is positive definite (even in unphysical regions) and furthermore can not approach zero faster than the nonplanar part;
- related processes always have the same decomposition;
- we do not even need a compact analytical expression for the matrix elements squared to carry out the colour decompositions.

This last feature becomes useful for more complex n-body processes,[4] where the most efficient method for calculating matrix elements may be to utilise the helicity amplitude formalisms [12, 13].

One drawback of this method is that the formulae may not always be aesthetically preferable to those obtained with the method of Marchesini and Webber, for example in the case of[5] $qq \to qq$. However, this is not a problem in computer simulations, which are the only circumstances where the methods described in this paper are put to practice.

4. The 2 parton $\to$ 2 sparton processes

The gauge invariance and the simplicity of these processes greatly compactify the expressions for matrix elements. In particular, as in the QCD case, all supersymmetric processes of the form $q\bar{q} \to gg$ (where q and g refer here to quarks or squarks and gluons or gluinos, respectively) can be expressed as

$$(\text{colour factor}) \times \frac{(u_4^2)_t + (t_3^2)_u + (-s^2/N_C^2)_{\text{n.p.}}}{s^2} \times \overline{|\mathcal{M}|^2}_{\text{QED}}\,, \tag{4.1}$$

provided that the sparticles exchanged in the t and the u channels have masses $m_{3(4)}$ and $m_{4(3)}$ respectively for the process $1+2 \to 3+4$. Here $u_4 = u - m_4^2 = -2p_1 \cdot p_4$ and $t_3 = t - m_3^2 = -2p_1 \cdot p_3$. When $m_t \neq m_{3(4)}$ or $m_u \neq m_{4(3)}$, as is the case generally in $q\bar{q} \to \tilde{g}\tilde{g}$, there are correction terms proportional to the differences in the squared masses.

The colour flows are identical to the ordinary QCD case shown in figs. 1–4.

[4] In [14] a variant of this procedure is adopted as the default in the process $e^+e^- \to q\bar{q}gg$.

[5] In fact, for all other massless QCD processes the procedure described here yields results identical to those in [1].

We present the spin and colour averaged squared matrix elements, general for any N_C and nondegenerate squark masses. These are divided by a statistical factor of two when the final state spartons are identical.

Apart from the colour structures, the formulae match those of [15] when $N_C = 3$, and those of tree-level expressions in [16] when left and right squark masses are taken to be degenerate. Stop and sbottom mixings are not considered here since we are dealing with chirality independent interactions.

In the formulae that follow, g_s is the strong coupling, evaluated at some scale which is not determined at tree level (in the HERWIG Monte Carlo it is taken to be $\sqrt{s}$). s is the effective centre-of-mass energy $s_{\text{eff}} = s_{\text{tot}} x_1 x_2$, t and u similarly. Since we are taking the initial state partons to be massless it follows that $s+t+u = m_3^2+m_4^2$ and $s+t_3+u_4 = 0$. Also $ut - m_3^2 m_4^2 = s p_T^2 \geq 0$, where p_T is the outgoing transverse momentum. $m_{\tilde{g}}$ is the gluino mass, $M_{i_{L,R}}$ are the i-th generation squark masses. Charge conjugate final states must also be included in simulations.

$$\overline{|\mathcal{M}|^2}(q_i\bar{q}_i \to \tilde{g}\tilde{g}) = \frac{g_s^4 C_F}{4} \sum_{L,R} C_{L,R} \tag{4.2}$$

with

$$\begin{aligned} C_{L,R} = \frac{2sp_T^2}{s^2} & \left[\frac{(u_4^2-\Delta^2)_t + (t_3^2-\Delta^2)_u - (s^2/N_C^2)_{\text{n.p.}}}{(u_4-\Delta)(t_3-\Delta)} \right] + \\ & + \Delta^2 \left[\left(\frac{1}{(t_3-\Delta)^2} \right)_t + \left(\frac{1}{(u_4-\Delta)^2} \right)_u - \frac{1}{N_C^2} \left(\frac{1}{t_3-\Delta} - \frac{1}{u_4-\Delta} \right)^2_{\text{n.p.}} \right], \end{aligned} \tag{4.3}$$

where $\Delta = M_{i_{L,R}}^2 - m_{\tilde{g}}^2$. $\sum_{L,R}$ denotes a summation over the left and right squarks. Terms marked with subscript t correspond to the colour flow $(1 \to 3, 3 \to 4, 4 \to 2)$ and those with subscript u correspond to the colour flow $(1 \to 4, 4 \to 3, 3 \to 2)$, as in fig. 3. Although it is possible to distribute the nonplanar terms between the two colour flows using the 'intuitive' method, for the sake of consistency we advocate the use of eq. (3.3).

$$\begin{aligned} \overline{|\mathcal{M}|^2}(gg \to \tilde{g}\tilde{g}) = \frac{g_s^4 N_C^2}{N_C^2-1} \cdot \frac{u_4 t_3}{2} & \left[u_4^2 + t_3^2 + \frac{4 m_{\tilde{g}}^2 s^2 p_T^2}{u_4 t_3} \right] \times \\ & \times \left[\left(\frac{1}{s^2 t_3^2} \right)_{st} + \left(\frac{1}{s^2 u_4^2} \right)_{su} + \left(\frac{1}{u_4^2 t_3^2} \right)_{ut} \right]. \end{aligned} \tag{4.4}$$

The colour flows are $(1 \to 3, 3 \to 4, 4 \to 2, 2 \to 1)$ for $1/s^2 t_3^2$, $(1 \to 4, 4 \to 3, 3 \to 2, 2 \to 1)$ for $1/s^2 u_4^2$, and $(1 \to 4, 4 \to 2, 2 \to 3, 3 \to 1)$ for $1/t_3^2 u_4^2$ (fig. 4). There is no nonplanar term, as any cross term between two colour flows is equivalent to the square of the other.

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$$\overline{|\mathcal{M}|^2}(gq_i \to \tilde{g}\tilde{q}_{i_{L,R}}) = \frac{g_s^4}{4}\left[-u_4 - 2(m_4^2 - m_3^2)\left(1 + \frac{m_3^2}{t_3} + \frac{m_4^2}{u_4}\right)\right] \times$$
$$\times \frac{(u_4^2)_s + (s^2)_u - (t_3^2/N_C^2)_{\text{n.p.}}}{st_3u_4} \,. \tag{4.5}$$

The colour flows are $(2 \to 1, 1 \to 3, 3 \to 4)$ for the s-channel term and $(2 \to 3, 3 \to 1, 1 \to 4)$ for the u-channel term (fig. 3). Equation (3.3) leads to the decomposition of the nonplanar term $t_3^2 = [(u_4^2 t_3^2)_s + (s^2 t_3^2)_u]/(u_4^2 + s^2)$.

$$\overline{|\mathcal{M}|^2}(q_iq_j \to \tilde{q}_{i_{L,R}}\tilde{q}_{j_{L,R}}) = \frac{g_s^4 C_F}{2N_C} \cdot \frac{m_{\tilde{g}}^2 s}{1+\delta_{ij}} \times$$
$$\times \left[\left(\frac{1}{(t-m_{\tilde{g}}^2)^2}\right)_t + \left(\delta_{ij}\frac{1}{(u-m_{\tilde{g}}^2)^2}\right)_u - \left(\delta_{ij}\frac{2/N_C}{(t-m_{\tilde{g}}^2)(u-m_{\tilde{g}}^2)}\right)_{\text{n.p.}}\right]. \tag{4.6}$$

The colour flows are $(1 \to 4, 2 \to 3)$ for the t-channel term and $(1 \to 3, 2 \to 4)$ for the u-channel term (fig. 2). For the case $i \neq j$, the colour flow is uniquely $(1 \to 4, 2 \to 3)$, as in fig. 1.

$$\overline{|\mathcal{M}|^2}(q_iq_j \to \tilde{q}_{i_{L,R}}\tilde{q}_{j_{R,L}}) = \frac{g_s^4 C_F}{2N_C} \cdot sp_T^2 \left[\left(\frac{1}{(t-m_{\tilde{g}}^2)^2}\right)_t + \left(\delta_{ij}\frac{1}{(u-m_{\tilde{g}}^2)^2}\right)_u\right]. \tag{4.7}$$

The colour flows are again $(1 \to 4, 2 \to 3)$ for the t-channel term and $(1 \to 3, 2 \to 4)$ for the u-channel term. There is no nonplanar term. For the case $i \neq j$, the colour flow is again uniquely $(1 \to 4, 2 \to 3)$.

$$\overline{|\mathcal{M}|^2}(q_i\bar{q}_j \to \tilde{q}_{i_{L,R}}\tilde{q}^*_{j_{L,R}}) = \frac{g_s^4 C_F}{2N_C} \cdot sp_T^2 \times$$
$$\times \left[\left(\frac{1}{(t-m_{\tilde{g}}^2)^2}\right)_t + \left(\delta_{ij}\frac{2}{s^2}\right)_s - \left(\delta_{ij}\frac{2/N_C}{s(t-m_{\tilde{g}}^2)}\right)_{\text{n.p.}}\right]. \tag{4.8}$$

The colour flows are $(1 \to 2, 4 \to 3)$ for the t-channel term and $(1 \to 3, 4 \to 2)$ for the s-channel term. For the case $i \neq j$, the colour flow is uniquely $(1 \to 2, 4 \to 3)$.

$$\overline{|\mathcal{M}|^2}(q_i\bar{q}_j \to \tilde{q}_{i_{L,R}}\tilde{q}^*_{j_{R,L}}) = \frac{g_s^4 C_F}{2N_C} \cdot \frac{m_{\tilde{g}}^2 s}{(t-m_{\tilde{g}}^2)^2} \,. \tag{4.9}$$

The colour flow is uniquely $(1 \to 2, 4 \to 3)$, regardless of i and j.

$$\overline{|\mathcal{M}|^2}(q_i\bar{q}_i \to \tilde{q}_{j_{L,R}}\tilde{q}^*_{j_{L,R}}) = \frac{g_s^4 C_F}{N_C} \cdot \frac{sp_T^2}{s^2} \,. \tag{4.10}$$

Here $i \neq j$. The colour flow is uniquely $(1 \to 3, 4 \to 2)$.

$$\overline{|\mathcal{M}|^2}(gg \to \tilde{q}_{i_{L,R}} \tilde{q}^*_{i_{L,R}}) = \frac{g_s^4 N_C}{2(N_C^2 - 1)} \left[(s p_T^2)^2 + m_3^2 m_4^2 s^2 \right] \times \\ \times \frac{(u_4^2)_t + (t_3^2)_u - (s^2/N_C^2)_{\text{n.p.}}}{s^2 t_3^2 u_4^2} . \quad (4.11)$$

The colour flows are $(4 \to 2, 2 \to 1, 1 \to 3)$ for the t-channel term and $(4 \to 1, 1 \to 2, 2 \to 3)$ for the u-channel term (fig. 3).

5. Conclusions

We have discussed the techniques for calculating the 'full terms' in QCD and supersymmetric QCD processes which involve multiple colour flows. The exact distribution of nonplanar terms between the full terms is expected to make little significant difference to the prediction of soft radiation patterns, but we argued that the conventional method used to achieve this is inadequate.

We have presented the formulae for 2 parton $\to$ 2 sparton processes, together with the colour flows associated with them. These are incorporated in the Monte Carlo event generator `HERWIG` 6.1, to be released shortly. However, the study of the showering and hadronisation of supersymmetric particles has not been carried out in sufficient detail, and it is clear that further investigations are necessary.

Acknowledgments

I thank Bryan Webber for suggesting the study of colour decompositions of supersymmetric processes to me and for the fruitful discussions, Stefano Moretti for continuous encouragement and for testing some of my earlier attempts at colour decompositions, and Pino Marchesini for helpful comments. I acknowledge the financial support from Trinity College and the Committee of Vice-Chancellors and Principals of the Universities of the UK.

References

[1] G. Marchesini and B.R. Webber, *Nucl. Phys.* **B 310** (88) 461.

[2] R.K. Ellis, G. Marchesini and B.R. Webber, *Nucl. Phys.* **B 286** (87) 643.

[3] B.R. Webber, talk presented at the Workshop on Theory of LHC Processes, CERN, 13 February 1998.

[4] S. Moretti, K. Odagiri, M.H. Seymour and B.R. Webber *Implementation of supersymmetric processes in the HERWIG event generator*, `Cavendish-HEP-98/06`, currently in preparation.

[5] G. Marchesini, B.R. Webber, G. Abbiendi, I.G. Knowles, M.H. Seymour and L. Stanco, *Comput. Phys. Commun.* **67** (92) 465.

[6] G. Corcella, I.G. Knowles, G. Marchesini, S. Moretti, K. Odagiri, M.H. Seymour and B.R. Webber, *HERWIG version 6.1, including supersymmetric processes in hadronic collisions*, `Cavendish-HEP-98/04`, currently in preparation.

[7] B.L. Combridge, J. Kripfganz and J. Ranft, *Phys. Lett.* **B 70** (77) 234.

[8] The CDF collaboration, *Phys. Rev.* **D 50** (94) 5562.

[9] The D0 collaboration, *Phys. Lett.* **B 414** (97) 419 [`hep-ex/970601`].

[10] G.R. Farrar, *Phys. Rev.* **D 51** (95) 3904 [`hep-ph/9407401`].

[11] G. Marchesini and B.R. Webber, *Nucl. Phys.* **B 330** (90) 261.

[12] K. Hagiwara and D. Zeppenfeld, *Nucl. Phys.* **B 274** (86) 1.

[13] H. Murayama, I. Watanabe and K. Hagiwara, *HELAS: HELicity Amplitude Subroutines for Feynman diagram evaluations*, KEK report 91–11, January 1992.

[14] I.G. Knowles, T. Sjostrand et al., *QCD event generators*, in the second volume of the Proceedings of the LEP2 Physics Workshop, Geneva, 15-16 June 1995.

[15] S. Dawson, E. Eichten and C. Quigg, *Phys. Rev.* **D 31** (85) 1581.

[16] W. Beenakker, R. Höpker, M. Spira and P.M. Zerwas, *Phys. Lett.* **B 349** (95) 463 [`hep-ph/9501292`].